On the Existence of Equilibrium Bank Runs in a Diamond-Dybvig Environment*

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Abstract

In a version of the Diamond and Dybvig [6] model with aggregate uncertainty, we show that there exists an equilibrium with the following properties: all consumers deposit at the bank, all patient consumers wait for the last period to withdraw, and the bank fails with strictly positive probability. Furthermore, we show that the probability of a bank failure remains bounded away from zero as the number of consumers increases. We interpret such an equilibrium as reflecting a bank run, defined as an episode in which a large number of people withdraw their deposits from a bank, forcing it to fail.

Our results show that we can have equilibrium bank runs with consumers poorly informed about the true state of nature, a sequential service constraint, an infinite marginal utility of consumption at zero, and without consumers' panic and sunspots. We therefore think that aggregate risk in Diamond-Dybvig-like environments can be an important element to explain bank runs.

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1 Introduction

Throughout history, we observe many episodes of a large number of people withdrawing their deposits from a bank, forcing it to fail. Since the work of Diamond and Dybvid [6], these *bank run*[1] episodes are understood as a consequence of an illiquid banking system: the main function of banks is to lend long and to borrow short, thus yielding an illiquid asset structure; this in turn creates the potential for bank runs.

Recently, Green and Lin [10, 11] have challenged this view, by showing that bank runs can be eliminated when banks use more sophisticated contracts in a Diamond and Dybvig environment. Thus, they conclude that the lack of liquidity highlighted by Diamond and Dybvig does not necessarily leads to bank runs. Green and Lin's result then gave rise to the view that the Diamond and Dybvig model is not appropriate to explain bank runs.

The main goal of our paper is to provide theoretical support for the Diamond and Dybvig model. Some support has already been given by Peck and Shell [16], who have shown that bank runs can occur due to sunspots in equilibrium under the optimal contract, even when banks are allowed to offer contracts similar to those in Green and Lin [11]. We provide additional support by showing that bank runs can occur with positive probability in equilibrium as a consequence of aggregate uncertainty and poorly informed consumers. Furthermore, we show that the probability of bank runs remains bounded away from zero as the size of the economy increases. This will allow us to conclude that the positive probability of bank runs is mainly driven by the desire to provide better risk sharing and more liquidity to consumers. Thus, bank runs in our framework are directly related with liquidity.

Our results also show that bank runs can be generated in the presence of many elements that make them costly to consumers and/or difficult to occur in equilibrium. These include:

[1] We follow Allen and Gale [2, p. 1245] in defining the notion of a bank run:

> "From the earliest times, banks have been plagued by the problem of *bank runs* in which many or all of the bank's depositors attempt to withdraw their funds simultaneously."

In particular, we will classify as a bank run an episode in which many depositors withdraw (enough to make the bank fail) regardless of whether or not some of them are panicking. Unfortunately, there is no consensus in the terminology; for instance, Peck and Shell [17] refer to the episodes we call bank runs as "no-run rationing" or "running out of funds".

1. absence of consumers' panic,
2. absence of sunspots,
3. absence of mixed strategies,
4. sequential service constraint (and so zero consumption to late withdrawers),
5. an infinite marginal utility of consumption at zero,
6. absence of information about the true state of the economy,
7. recoverability of investment in the productive technology, and
8. a non-negligible probability of a high fraction of impatient consumers occurring.

This suggests that bank runs are quite robust in our Diamond and Dybvig framework.

Before we discuss each of the above elements and the reason why they cause difficulties, we will present our results in more detail. We consider an environment similar to the one in Diamond and Dybvig [6]. In particular, consumers can be of two distinct types: impatient consumers, who need to consume early, and patient consumers, willing to postpone consumption. In contrast with them, we assume that there is a finite number of consumers. We assume that there is aggregate uncertainty, modelled in the following way: first, the probability of each consumer being impatient is chosen according to a continuous density function; then, the consumers' type is determined in an i.i.d. way. As in Diamond and Dybvig, the consumers' type is their own private information. Furthermore, we assume that although each consumer knows his type, he does not know the true value of the probability of each consumer being impatient.

Consumers can deposit their initial endowment in a bank, or invest it directly. The banking system is assumed to be competitive and so banks offer contracts to depositors in order to maximize their ex-ante welfare.

Regarding the contracts that banks offer to consumers, we depart from the optimal contracting approach of Green and Lin [10, 11], by following their suggestion in [11, p. 24]. According to them, the incentive problems of banking executives may be missing in their model. This may explain "(...)

why the banking contract in [their] model is not observed and why runs have historically occurred." In the contract proposed by Green and Lin, the amount paid by the bank to an early depositor depends on the information the bank has obtained from those who have already contacted it. However, as Green and Lin point out, if each depositor does not have access to that information, such contract is infeasible. Following this view, we assume that the only information depositors have, besides their own type, is whether the bank has failed or not. Specifically, we assume that a monetary authority makes a public announcement whenever the bank fails and so, the bank can make payments contingent on whether it has failed (in which case it pays nothing) or not. However, the bank cannot make payments contingent on any other variable. Thus, we are taking Green and Lin's suggestion to the extreme, and, in fact, returning to the same type of contract considered originally by Diamond and Dybvig.[2] We want to emphasize that this is an assumption on the technology to enforce contracts: although a contract between the bank and the deposits that makes payments contingent on the number of people who have already withdrawn can be verifiable by a third party, we assume that those verification costs are too high to make them efficient.

Our main result then shows that if a sufficiently high fraction of impatient consumers is possible to occur[3] and if the number of consumers is sufficiently high, then there exists an equilibrium in which:

1. every consumer deposits,
2. no patient consumer withdraws early and,
3. the bank fails with a strictly positive probability.

In the above equilibrium, the bank failure corresponds to a situation where the fraction of impatient consumers is high. In such a case, we would observe a large number of depositors trying to withdraw, which would force the bank to fail — a situation that corresponds to a bank run.

[2] We assume that consumers are isolated from each other, which implies the same trading restrictions (i.e., that depositors cannot sell their position at the bank) as in Diamond and Dybvig [6]. The importance of these trading restrictions has been analyzed by Jacklin [14].

[3] i.e., if the support of density function determining the probability of each consumer being impatient contains values sufficiently close to one.

There is a sense in which our result above is weak: if the probability of a consumer being impatient is positive, then the probability that all consumers are impatient is also positive. This implies that all that is required for a positive probability of a bank run is that the bank offers an interest rate greater than one. Thus, the only way for the bank to avoid a bank run is to offer a contract in which there is absolutely no risk sharing. Our second main result strengthens the above result by showing, under the same assumptions, that the probability of a bank run is bounded away from zero.

The probability of a bank run, and the fact that it is bounded away from zero, depends on how illiquid the bank is. The reason is better understood by considering large economies, in which we are mainly interested. By the law of large numbers, as the number of consumers increases, the fraction of impatient depositors in the population converges to the probability of each one being impatient. In the limit, if the support of the density function determining the probability of each consumer being impatient is $[\underline{t}, \bar{t}]$, the bank will fail with a positive probability whenever it offers an interest rate higher than $\bar{r} = 1/\bar{t}$. Thus, we can rephrase our second main result as showing that the interest offered by the bank is higher and bounded away from $\bar{r}$. The main intuition behind it is that banks find it optimal to offer an interest rate higher than $\bar{r}$ in order to provide better risk sharing to consumers. The fact that the interest rate offered by the bank is bounded away from $\bar{r}$ then leads to a probability of bank run that is bounded away from zero.

Regarding the main results of the paper, our inspiration came from Wallace [20, p. 12], where he writes: "In my model, the cause of a bank run and a partial suspension is exogenous — an aggregate shock to tastes that makes the number of people wanting to withdraw unusually large." When we combine this insight with the simple contracting approach, we find that the bank prefers to face a positive probability of a bank run, and a consequent failure, in order to provide better risk sharing to its depositors. As suggested by Wallace, the bank run will occur when the number of impatient consumers is high. Our results have, nevertheless, two important differences compared with Wallace's: first, bank runs have more severe consequences in our results, since they cause banks to fail, whereas in Wallace's they lead to a partial suspension; second, Wallace's result relies on the assumption that there is a small amount of aggregate risk limited to a small group of individuals, which is not needed in ours — we believe that our result allows us to think of bank runs as a large-scale, society-wide phenomenon.[4]

Although our results are intuitive, they are not easy to establish. Part of the difficulty arises because we allow for several elements that make them hard to occur in equilibrium. First, since all patient consumers prefer to wait, there is no consumers' panic in the bank runs of our model, an element that was important in Diamond and Dybvig [6] and Peck and Shell [16], among others. Second, our results do not require sunspots, which are a crucial element for Peck and Shell's result. Third, we restrict consumer choice to pure strategies; without this assumption one can show the existence of a positive probability of a bank run as in Adão and Temzelides [1]. Fourth, we require banks to satisfy a sequential service constraint (which implies that, in a bank run, late withdrawers will receive zero consumption). Fifth, we allow for an infinite marginal utility of consumption at zero, which together with the previous assumptions makes bank runs costly to those consumers who are unable to withdraw. Sixth, we assume that consumers have no information about the true state of the economy, and seventh, that the investment in the productive technology can be recovered — without these two assumptions, one can generate bank runs in which consumers run when the state of the economy is bad, as in Allen and Gale [2]. Finally, we allow for a non-negligible probability of a high fraction of impatient consumers occurring; this increases the risk of a bank run, which is costly.

Furthermore, our results hold for (large, but) finite economies, and not merely in a limit or in a continuum of agents economy. Thus, although we infer them from the study of a limit economy, they cannot be regarded as an artifact of an infinite population.[5]

In conclusion, we have shown that within the standard Diamond and Dybvig [6] environment, bank runs can be explained as the result of aggregate uncertainty, simple contracting, and without many of the elements emphasized in the literature. Furthermore, in our framework, bank runs are intimately linked with the consumer's desire for liquidity, an idea expressed originally by Diamond and Dybvig.

[4]See Peck and Shell [17] and Ennis and Keister [7] for a similar point.

[5]See Barlo and Carmona [4], Carmona [5] and Meirowitz [15] for more on this issue.

2 The model

In this section, we formally describe the model. The model presented here is similar to that presented in Diamond and Dybvig [6].

There are three periods $T = 0, 1, 2$. There is a single consumption good. There are two technologies: first, the consumer good can be stored from one period to the other; the gross rate of return equals 1. Second, there is a productive technology, by which one unit invested in period 0 yields R, with $R > 1$, units of consumption in period 2; furthermore, if the investment is interrupted in period 1, it will yield one unit (i.e., the investment can be recovered).

There is a finite number of consumers denoted by $n \in \mathbb{N}$. All consumers are identical in period 0. Consumers receive an endowment of one unit of consumption good in period 0 and zero in the remaining periods.

Each consumer can be of two distinct types, denoted by type 1 and type 2. A type 1 consumer values consumption in period 1 only (impatient consumer), whereas a type 2 consumer values consumption only in period 2 (patient consumer).

In period 1, nature draws a type for every consumer in the following way: first, a number in $t \in [0, 1]$ is drawn according to a probability measure μ; then each consumer's type is drawn in an i.i.d. way with a probability t of being of type 1. It follows that the fraction of impatient consumers equals $i \in S_n = \{k/n : k = 0, \ldots, n\}$ with probability $p_{n,t}(i) = t^{ni}(1-t)^{n-ni}$ when there are n consumers and the probability of each being a type 1 is t.

Each consumer knows his own type, but not the type of the others; i.e., consumers' type is their own private information. Furthermore, no consumer knows the realized value of t.

We make the following assumptions on the uncertainty:

Assumption 1 *1. The support of μ is an interval contained in $[0, t']$, where $0 < t' \leq 1$;*

2. there exists a continuous function $f : \mathrm{supp}(\mu) \to \mathbb{R}_+$ such that $\mu(B) = \int_B f$ for every Borel measurable set $B \subseteq [0, 1]$.

The continuity of the density f implies that if $\bar{t} = \max\{t : t \in \mathrm{supp}(\mu)\}$, then $F(t) = \int_0^t f < 1$ for all $t < \bar{t}$. Given any $\bar{t} \in [0, 1]$, we write $f \in \mathcal{F}_{\bar{t},B,x}$ when f is continuous, bounded by $B > 0$, $\bar{t} = \max\{t : t \in \mathrm{supp}(\mu)\}$ and $x = \int_0^1 t f(t)\mathrm{d}t$.

Let c_1 denote the individual consumption received by a consumer in period 1, let c_2 denote individual consumption received in period 2, and let Θ be the type of the agent. The utility derived by every agent from the consumption of the bundle (c_1, c_2) is

$$U(c_1, c_2, \Theta) = \begin{cases} u(c_1) & \text{if } \Theta = 1, \\ u(c_1 + c_2) & \text{otherwise,} \end{cases} \tag{1}$$

where $u : \mathbb{R}_+ \to \mathbb{R}$ is twice continuously differentiable, strictly increasing and strictly concave. Furthermore, we assume:

Assumption 2 *1.* $-cu''(c)/u'(c) > 1$ *for* $c \geq 1$*;*

2. $u(0) = 0$*;*

3. $\lim_{c \to 0} u'(c)c \in \mathbb{R}$.

Every agent is assumed to maximize the ex-ante (relative to period 0) expected utility $E[U(c_1, c_2, \Theta)]$.[6]

We next describe the banking industry. There is a representative bank behaving competitively. The bank offers to the depositors a contract specifying a fixed claim of r per unit deposited to agents withdrawing in period 1. The bank is mutually owned, and it is liquidated in period 2. This implies that period 2 withdrawers will share the remainder of the bank's assets equally among themselves.

If and when the bank fails, consumers will be informed (say, by a regulatory entity). The bank is closed and the remaining assets are distributed in period 2 to those claiming it. We assume that no impatient consumer will claim anything in period 2; in any case, an impatient consumer is indifferent between claiming or not, and so we can justify this assumption by postulating a positive cost of exercising it.

[6] An example of a function satisfying all the above assumptions is $u(c) = -\exp^{-2c} + 1$. In this case, we would have $\lim_{c \to 0} u'(c) = 2$. An example of a function satisfying $\lim_{c \to 0} u'(c) = \infty$ is

$$u(c) = \begin{cases} \sqrt{c} & \text{if } c \leq 1/2, \\ -\frac{\sqrt{2}c^{-1}}{4} + \sqrt{2} & \text{otherwise.} \end{cases} \tag{2}$$

This function satisfies all the assumptions except that it does not have a second derivative at 1/2. Nevertheless, all our results extend to the case in which u'' is continuous for all $c \geq 1$.

Thus, the amounts received by depositors are as follows: Let A denote the total amount deposited in period 0 and r be the interest rate offered by the bank. Consider a depositor j willing to withdraw in period 1 and let f_j denote the number of withdrawers arriving at the bank before consumer j. Then j's period 1 payoff is equal to r if $rf_j \leq A$ and 0 otherwise. If a depositor waits for period 2 to withdraw, then he receives

$$\max\left\{\frac{R(A-rf)}{1-f}, 0\right\}, \tag{3}$$

where f denotes the fraction of the depositors who have withdrawn in period 1.

We assume that consumers are isolated from each other during period 1, although each one contacts the bank at some point in that period. As Wallace [19] has shown, this implies that the bank has to satisfy a sequential service constraint; that is, the bank must serve the depositors withdrawing in period 1 in the (random) order that they arrive at the bank until it runs out of assets. We assume that all orderings are equally likely, and so each occurs with a probability of $1/m!$ when there are $0 \leq m \leq n$ withdrawers. In order to evaluate different strategies, each consumer needs to know the probability of arriving at the bank before it fails. If the bank can fully pay k depositors, then the probability that a given consumer is fully paid equals $\min\{1, k(m-1)!/m!\} = \min\{1, k/m\}$. We will write this probability as $\alpha(m/n, k/n)$, that is, we use as arguments the fraction of depositors trying to withdraw and the fraction of depositors that can be fully paid in period 1. In general, this probability depends on the interest rate r offered by the bank and on the strategies chosen by the other consumers. We let $k_n(r)$ satisfy $rk_n(r) \leq 1$ and $r(k_n(r) + 1/n) > 1$; thus, $nk_n(r)$ is the number of depositors that can be fully paid in period 1 when the bank offers r.

A strategy of the bank is the choice of $r \in [1, R]$; the bank chooses r in order to maximize ex-ante utility of the consumers (recall that they are equal ex-ante). This behavior is motivated by the competitive nature of the banking industry.[7] More generally, we could assume that the bank offers $r \in \mathbb{R}$, but risk-aversion will ensure that $r \in [1, R]$. The bank chooses the interest rate in period 0 and then announces it to the consumers.

[7] Adão and Temzelides [1] have shown in a similar framework that another type of bank's behavior is possible in equilibrium; however, bank's maximization of the ex-ante utility of the consumers is the only behavior plausible in more refined notions of equilibrium.

In period 0, each consumer will choose whether to deposit given the interest rate offered by the bank; those that do not deposit will invest in the productive technology. For simplicity, we assume that consumers have to deposit all of the endowment. In period 1 each consumer learns her type and then chooses either to withdraw from the bank or to wait, depending on her type, on the interest rate and on her deposit choice. Consumers that withdraw their deposit in period 1 can either consume the goods received or store them.[8] Hence, a strategy for a consumer is (d, w), where d is a function from $[1, r]$ into $\{0, 1\}$, and w is a function from $[1, R] \times \{0, 1\} \times \Theta$ into $\{0, 1\}$. We make the convention that $d(r) = 1$ stands for the choice of depositing, and similarly $w(r, d, \Theta) = 1$ means that she will withdraw in period 1.[9]

A *symmetric equilibrium* is then r^*, d^*, and w^* such that $w^*(r, d, \Theta)$ is optimal for all (r, d, Θ), $d^*(r)$ is optimal for all r, and r^* is optimal taking as given agents' strategies.

The *bank fails in the first period* if $d^*(r^*) = 1$ and $1 < r^*m/n$ where r^* is an equilibrium interest rate and m is the number of depositors that choose to withdraw (i.e., $w^*(r^*, 1, \Theta) = 1$). That is, the total value of assets that depositors plan to withdraw in period 1 strictly exceeds the total value of assets owned by the bank, also in period 1.

We say that *an equilibrium bank run without consumers' panic occurs asymptotically with positive probability* if there exists $N \in \mathbb{N}$ such that $n \geq N$ implies the existence of a symmetric equilibrium with the following properties:

1. every consumer deposits (i.e., $d^*(r^*) = 1$),

2. all patient consumers wait (i.e., $w^*(r^*, d^*(r^*), 2) = 0$),

3. the bank fails in the first period with strictly positive probability.

In the above equilibrium there is a bank run in the sense that a large number of depositors go to the bank, causing it to fail. However, since all patient consumers prefer to wait, we say that there is no consumers' panic.

[8]This implies that storage will only be helpful for patient consumers who withdraw early.

[9]Note that we only allow pure strategies. It is important to note that it is possible to have bank runs in a mixed strategy equilibrium, as shown by Adão and Temzelides [1].

3 Equilibrium Bank Runs

In this section, we study whether equilibrium bank runs exist and how robust they are. In Section 3.1, we give a sufficient condition that guarantees the existence of equilibrium bank runs with positive probability in large economies. Then, in Section 3.2, we show that under the same assumption, the probability of bank runs is bounded away from zero as the number of consumers increases.

3.1 Existence of Equilibrium Bank Runs

In an equilibrium bank run, all consumers choose to deposit and patient consumers prefer to wait. So, it is necessary to guarantee the existence of equilibria with these properties, which one can do under the assumptions made in Section 2.

We can construct such an equilibrium by studying a particular maximization problem. This problem consists of choosing an interest rate r in order to maximize consumers' ex-ante utility among those that make consumers prefer to deposit and impatient consumers to wait, given that everyone is depositing and every patient depositor is waiting.

We describe this problem below which is indexed by the number of consumers. The ex-ante expected utility if all follow the above strategy can be obtained as follows: if a consumer is of type 1, then he receives r with probability $\alpha(i, k_n(r))$ when the fraction of impatient consumers is i and the bank can fully pay $nk_n(r)$ consumers. If the consumer is of type 2, he receives $\max\{0, \frac{R(n-ri)}{n-i}\}$. Thus, letting $U_n(r)$ denote the ex-ante expected utility, we have

$$\begin{aligned} U_n(r) = \int_0^1 f(t) \Bigg[& t \sum_{i \in S_{n-1}} p_{n-1,t}(i) \alpha\left(\frac{(n-1)i+1}{n}, k_n(r)\right) u(r) \\ & +(1-t) \sum_{i \in S_{n-1}} p_{n-1,t}(i) u\left(\max\left\{0, \frac{R(n-r(n-1)i)}{n-(n-1)i}\right\}\right) \Bigg] \mathrm{d}t. \end{aligned} \tag{4}$$

In particular, suppose that the bank offers an interest rate equal to r, all consumers deposit, and all patient consumers wait for the second period to withdraw. Then, the expected utility for a patient consumer equals

$$\int_0^1 f(t) \sum_{i \in S_{n-1}} p_{n-1,t}(i) u \left(\max \left\{ 0, \frac{R(n - r(n-1)i)}{n-(n-1)i} \right\} \right) \mathrm{d}t. \tag{5}$$

If one patient consumer decides to withdraw in period 1, then his expected utility, when all the other patient consumers withdraw in period 2, is given by

$$\int_0^1 f(t) \sum_{i \in S_{n-1}} p_{n-1,t}(i) \alpha \left(\frac{(n-1)i+1}{n}, k_n(r) \right) u(r) \mathrm{d}t. \tag{6}$$

Thus, letting

$$W_n = \{ r \in [1, R] : WL_n(r) \geq WR_n(r) \}, \tag{7}$$

where $WL_n(r)$ is defined by equation (5) and $WR_n(r)$ by equation (6), we have that any patient consumer will choose to withdraw in period 2 provided that r belongs to W, all consumers deposits, and all other patient consumers wait for period 2 to withdraw.

If a consumer decides not to deposit, then his ex-ante expected utility is simply

$$\int_0^1 f(t) \left[tu(1) + (1-t)u(R) \right] \mathrm{d}t, \tag{8}$$

which is equal to $U_n(1)$. Hence, letting

$$D_n = \{ r \in [1, R] : U_n(r) \geq U_n(1) \}, \tag{9}$$

we see that any consumer will choose to deposit provided that r belongs to D, all other consumers deposit, and all patient consumers wait for period 2 to withdraw.

Thus, consider the following problem:

$$\begin{aligned} &\max_{r \in [1,R]} U_n(r) \\ &\text{subject to } r \in W_n \cap D_n, \end{aligned} \tag{10}$$

One can show that this problem has a solution (see Lemma 11 in the Appendix). It is then easy to construct an equilibrium in which all consumers deposit and all patient consumers wait (see Lemma 12).

Although the existence of such equilibria is necessary for our purposes, it is not enough. Without further assumptions, the interest rate offered by the bank in such equilibria may be equal to one, in which case there will be no bank run. Thus, we need additional assumptions in order to guarantee that bank runs occur with positive probability. Essentially, we need the support of f to have values sufficiently close to one, and a large population.

Proposition 1 *For every $B > 0$ and $x \in (0,1)$ there exists $\tau \in [0,1]$ such that if $\tau < \bar{t} < 1$ and $f \in \mathcal{F}_{\bar{t},B,x}$ then an equilibrium bank run without consumers' panic occurs asymptotically with positive probability.*

One crucial element needed for Proposition 1 is a large population. By the law of large numbers, the fraction of impatient consumers in the population converges to the probability of each consumer being impatient as the size of the population increases. This implies that in the limit there is only one source of aggregate uncertainty, which is the one represented by f. This is in contrast with what happens in any finite economy, where some aggregate uncertainty stems from the consumers' idiosyncratic preference shocks. This is important partly because the probability that the fraction of impatient consumers is between $\bar{t}$ and 1 is always positive in any finite economy, but is zero in the limit.

The above comment suggests that the analysis of a limit problem where the law of large numbers holds might be useful and easier. In the Appendix A.1 we show that problem (10) converges, in the sense that all the functions involved converge uniformly, to the following problem:

$$\max_{r\in[1,R]} U(r) = \int_0^1 f(t)\left[t\alpha(t,k(r))u(r) + (1-t)u\left(\max\left\{0, \frac{R(1-rt)}{1-t}\right\}\right)\right] \mathrm{d}t \tag{11}$$

subject to $r \in W \cap D$,

where $k(r) = 1/r$, $D = \{r \in [1,R] : U(r) \geq U(1)\}$ and

$$W = \left\{r \in [1,R] : \int_0^1 f(t)u\left(\max\left\{0, \frac{R(1-rt)}{1-t}\right\}\right)\mathrm{d}t \geq \int_0^1 f(t)\alpha(t,k(r))u(r)\mathrm{d}t\right\}. \tag{12}$$

In this problem, the variable t can be thought of as the fraction of impatient consumers in the population. In this way, it is very similar to the one considered initially by Diamond and Dybvig [6].

The analysis of the limit problem above reveals that its solution exceeds $\bar{r} = 1/\bar{t}$, provided that $\bar{t}$ is sufficiently close to 1. This allows us to conclude that the solution to problem (10) is also greater than $\bar{r}$ if the population is large enough, leading directly to a positive probability of bank runs. This reasoning illustrates why we need the support of f to have values sufficiently close to one.

3.2 A Limit Result on the Probability of Bank Runs

As we have pointed out above, there are two sources of aggregate uncertainty in a finite economy. This implies that there are two reasons for having a positive probability of bank runs whenever the bank offers an interest rate greater than 1. First, for any possible $t > 0$, the probability that the fraction of impatient consumers is greater than $1/r$ is always positive — this probability has to do with the distribution associated with $p_{n,t}$. A second reason has to do with the distribution associated with f and which can also (by the law of large numbers) be thought of as representing the probability that the fraction of impatient consumers is greater than $1/r$.

Clearly, the second effect alluded to above can only take place if r exceeds $\bar{r}$. If this is not the case, then the probability of bank runs is essentially due only to the consumers' idiosyncratic shocks, and would vanish as the population size increases. What Proposition 2 below shows is that this is not the case: in fact, it shows that part of a positive probability of bank runs comes from the desire to provide a better risk sharing, which is expressed in an interest rate greater and bounded away from $\bar{r}$.

We now turn to Proposition 2. Under the conditions of Proposition 1, we know that asymptotically there exist equilibrium bank runs. For every n sufficiently large, let (r_n^*, d_n^*, w_n^*) be an equilibrium in which there is a bank run without consumers' panic. Let γ_n be the corresponding probability of a bank run.

Proposition 2 *The sequence $\{\gamma_n\}_n$ is bounded away from 0.*

At this point we can provide an easy illustration of why it is necessary that the support of f has values sufficiently close to one: if $R = 2$ and $\bar{t} = 1/3$, then $\bar{r} = 3 > R$. Thus, an upper bound for γ_n is obtained when the bank offers $r_n = R$ for all n, which we denote by $\tilde{\gamma}_n$. It follows easily from the law of large numbers that in this case $\tilde{\gamma}_n$ converges to zero. Thus, we need $\bar{r}$ to be large.

4 Concluding Remarks

We used the standard Diamond and Dybvig [6] framework to show that bank runs can be explained as the result of aggregate uncertainty and without many of the elements emphasized in the literature. In our version of the Diamond and Dybvig framework, bank runs will occur whenever there is a large number of depositors in need of short-term funds. Such bank runs are possible because banks will choose to offer a high short-term interest rate in order to provide better risk sharing for their depositors; however, the interest rate offered is so high that it leads to a positive probability that the bank will not have enough funds to pay all early withdrawers. Hence, the above bank runs are a direct consequence of the degree of banks' liquidity. Furthermore, our construction is such that the probability of a bank run is sufficiently small to guarantee that those who do not need funds early will prefer not to withdraw early. We thus depart from the idea that in a bank run some depositors withdraw when they do not need — this is the sense in which such runs involve no consumers' panic.

The type of equilibria on which we concentrate reflects some practical features of the banking system: banks offer liquidity and risk sharing to depositors, people deposit and withdraw only when they have to, and sometimes banks fail. Furthermore, an equilibrium of this type has the property of having no consumer panic, and still bank runs occur with a positive, bounded away from zero, probability.

Although not explicitly modelled, we interpret the aggregate uncertainty over the number of early withdrawers as reflecting business cycle conditions. For instance, we expect the number of people who need short-term funding to be influenced by the unemployment rate — this will be the case as long as unemployed individuals try to compensate the loss of income by using their assets to smooth out consumption. We then expect that fundamental shocks that lead to a large number of early withdrawers can create bank runs. Therefore, our results are consistent with the business cycle view of bank runs, a view that has received some empirical support (see Gorton [9] and Kaminsky and Reinhart [12]). This suggests that explicitly introducing the type of fundamental shocks studied in the business cycle literature might be promising, a challenge we take up in Amaral and Carmona [3].

A Appendix

In this appendix, we will prove our results. In Section A.1, and following Hildenbrand [13], we will start by showing that problem (10) converges to the type of problem studied by Diamond and Dybvig [6] as the number of consumers goes to infinity. Although we are interested in finite economies, the latter problem is interesting because it is easy to study. This is essentially because the law of large numbers holds.

In Section A.2, we study the limit problem. The main result there (Lemma 7) shows that the interest rate offered by the bank is sufficiently high to allow for bank runs in the limit problem under certain conditions.[10] The main difficulty regarding this result has to do with the possibility that the marginal utility of consumption is not bounded above; therefore, much of the effort is devoted to showing that standard limit results, such as the Lebesgue's dominated convergence theorem is still applicable.

In Section A.3, we show that for every $n \in \mathbb{N}$ (n denotes the population size) there exists an equilibrium in which all consumers deposit at the bank and all patient consumers wait to withdraw. Finally, and under the same conditions as in Lemma 7, we show in Section A.4 that there is a positive probability of an equilibrium bank run in large finite economies; furthermore, we show in Section A.5 that such a probability of an equilibrium bank run remains bounded away from zero as the number of consumers increases.

A.1 The Limit Problem

Lemma 1 *For any continuous function $h : [\alpha, \beta] \times [a, b] \to \mathbb{R}$, with $[a, b] \subseteq [0, 1]$,*

$$\int_{[a,b]} h(r, i) \mathrm{d}p_{n,t}(i) \to h(r, t)$$

uniformly in r and t.

Proof. Let $\varepsilon > 0$. Since h is continuous in $[\alpha, \beta] \times [a, b]$, a compact set, h is bounded. Let $B > \varepsilon$ be such that $|h| \leq B$ and let $\delta > 0$ be such that $|x - y| < \delta$ implies that $|h(x) - h(y)| < \varepsilon/2$. Then, since $p_{n,t}(B_\delta(t)) \geq 1 - \frac{t(1-t)}{n\delta^2}$

[10] For an illustration of the idea of Lemma 7, see the last column of Table 1 in Ennis and Keister [7].

(see Freund [8, p. 190]), it follows that

$$\begin{aligned}\left|\int_{[a,b]} h(r,i)\mathrm{d}p_{n,t}(i) - h(r,t)\right| \leq & \left|\int_{B_\delta(t)} h\mathrm{d}p_{n,t} - p_{n,t}(B_\delta(t))h(r,t)\right| + \\ & \left|\int_{B^c_\delta(t)} h\mathrm{d}p_{n,t} - p_{n,t}(B^c_\delta(t))h(r,t)\right| \leq \\ & \left(1 - \frac{t(1-t)}{n\delta^2}\right)\frac{\varepsilon}{2} + \frac{t(1-t)}{n\delta^2}2B \leq \\ & \left(1 - \frac{1}{4n\delta^2}\right)\frac{\varepsilon}{2} + \frac{1}{4n\delta^2}2B < \varepsilon,\end{aligned} \tag{13}$$

if n is sufficiently large. Thus, $\sup_{(r,t)} |\int_{[a,b]} h(r,i)\mathrm{d}p_{n,t}(i) - h(r,t)| \leq \varepsilon$ for n sufficiently large, which completes the proof. ■

Lemma 2 *Let h and h_n, for all $n \in \mathbb{N}$, be real-valued functions on $[\alpha, \beta] \times [a,b]$, with $[a,b] \subseteq [0,1]$, satisfying h is continuous and h_n converges uniformly to h. Then,*

$$\int_{[a,b]} h_n(r,i)\mathrm{d}p_{n,t}(i) \to h(r,t)$$

uniformly on r and t.

Proof. Let $\varepsilon > 0$. Then, since

$$\left|\int_{[a,b]} h_n(r,i)\mathrm{d}p_{n,t}(i) - \int_{[a,b]} h(r,i)\mathrm{d}p_{n,t}(i)\right| \leq \int_{[a,b]} |h_n - h|\mathrm{d}p_{n,t} \leq \sup_{(r,i)} |h_n(r,i) - h(r,i)|, \tag{14}$$

it follows by lemma 1 that

$$\begin{aligned}\left|\int_{[a,b]} h_n(r,i)\mathrm{d}p_{n,t}(i) - h(r,t)\right| \leq & \left|\int_{[a,b]} h_n(r,i)\mathrm{d}p_{n,t}(i) - \int_{[a,b]} h(r,i)\mathrm{d}p_{n,t}(i)\right| + \\ & \left|\int_{[a,b]} h(r,i)\mathrm{d}p_{n,t}(i) - h(r,t)\right| < \varepsilon,\end{aligned} \tag{15}$$

if n is sufficiently large. Thus, $\sup_{(r,t)} |\int_{[a,b]} h_n(r,i)\mathrm{d}p_{n,t}(i) - h(r,t)| \leq \varepsilon$ for n sufficiently large, which completes the proof. ■

Define for $t, i \in [0,1]$.

$$V^1_n(r,i) = \alpha\left(i + \frac{1-i}{n}, k_n(r)\right)u(r), \tag{16}$$

and

$$V_n^2(r,i) = u\left(\max\left\{0, \frac{R\left(1 - r\left(1-\frac{1}{n}\right)i\right)}{1-\left(1-\frac{1}{n}\right)i}\right\}\right). \tag{17}$$

Also let $k(r) = 1/r$,

$$V^1(r,i) = \alpha(i, k(r))u(r), \tag{18}$$

and

$$V^2(r,i) = u\left(\max\left\{0, \frac{R(1-ri)}{1-i}\right\}\right). \tag{19}$$

Lemma 3 *1. $k_n(r)$ converges to $k(r)$ uniformly;*

2. $\alpha(i+(1-i)/n, k_n(r))$ converges to $\alpha(i, k(r))$ uniformly in $i \in [0,1]$ and $r \in [1, R]$;

3. $tV_n^1(r,i)+(1-t)V_n^2(r,i)$ converges uniformly to $tV^1(r,i)+(1-t)V^2(r,i)$ in $i \in [0, 1-\varepsilon]$ and $r \in [1, R]$ for all $\varepsilon > 0$;

4. $(r,i) \mapsto tV^1(r,i) + (1-t)V^2(r,i)$ is continuous when $i \in [0, 1-\varepsilon]$ and $r \in [1, R]$ for all $\varepsilon > 0$ and $t \in [0,1]$.

Proof. 1. We have that

$$k(r) - \frac{1}{n} \le k_n(r) \le k(r), \tag{20}$$

which implies that $|k_n(r) - k(r)| \le 1/n$ and so, $k_n(r)$ converges uniformly to $k(r)$. Equation (20) can be established as follows: if $k_n(r) > k(r)$, then $rk_n(r) > rk(r) = 1$, a contradiction; if $k_n(r) < k(r) + 1/n$, then $r(k_n(r) + 1/n) < rk(r) = 1$, a contradiction.

2. We start by noting the following fact: if $(r,i) \mapsto a_n(r,i)$ converges uniformly (in i and r) to $(r,i) \mapsto a(r,i)$, $(r,i) \mapsto b_n(r,i)$ converges uniformly to $(r,i) \mapsto b(r,i)$, both a and b are bounded and b and b_n are bounded away from zero (i.e., there is $\eta > 0$ such that $b(r,i) \ge \eta$ and $b_n(r,i) \ge \eta$ for all (r,i) and all n), then a_n/b_n converges uniformly to a/b.

Suppose that $i \ge 1/r$. Then $\alpha(i, k(r)) = 1/(ri)$ and since $k(r) \ge k_n(r)$ and $i + (1-i)/n \ge i$, it follows that $\alpha(i+(1-i)/n, k_n(r)) = k_n(r)/(i+(1-i)/n)$. In order to apply the above fact, let $a_n(r,i) = k_n(r)$, $a(r,i) = 1/r$, $b_n(r,i) = i + (1-i)/n$ and $b(r,i) = i$. Since all the conditions are satisfied (in particular, $b_n(r,i) \ge b(r,i) \ge 1/R$), it follows that $\sup_{(r,i):ri\ge 1} |\alpha(i+(1-i)/n, k_n(r)) - \alpha(i, k(r))|$ converges to zero.

Finally, suppose that $i < 1/r$. Then, $k(r) = 1$ and so

$$\begin{aligned}
&|\alpha(i + (1-i)/n, k_n(r)) - \alpha(i, k(r))| = 1 - \min\left\{1, \frac{k_n(r)}{i + \frac{1-i}{n}}\right\} \leq \\
&1 - \frac{k_n(r)}{1 + 1/n} \leq 1 - \frac{k(r) - 1/n}{1 + 1/n} = 1 - \frac{1 - 1/n}{1 + 1/n}.
\end{aligned} \tag{21}$$

Thus, $\sup_{(r,i):ri<1} |\alpha(i + (1-i)/n, k_n(r)) - \alpha(i, k(r))| \to 0$. Hence, $\alpha(i + (1-i)/n, k_n(r))$ converges to $\alpha(i, k(r))$ uniformly in r and i.

3. It follows from part 2 that $V_n^1(r, i)$ converges uniformly to $V^1(r, i)$.

It remains to show that $V_n^2(r, i)$ converges uniformly to $V^2(r, i)$ if $r \in [1, R]$ and $i \in [0, 1 - \varepsilon]$, with $\varepsilon > 0$. Define

$$\beta_n(r, i) = \max\left\{0, \frac{R(1 - r(1 - \frac{1}{n})i)}{1 - (1 - \frac{1}{n})i}\right\}$$

and

$$\beta(r, i) = \max\left\{0, \frac{R(1 - ri)}{1 - i}\right\}.$$

Since $\beta_n(1, i) = \beta(1, i) = R$, we may assume that $r > 1$. Note also that $\beta_n(r, i) \geq \beta(r, i)$.

If $i \leq 1/r$ then $\beta(r, i) \geq 0$, and we readily see that all the conditions of the fact in part 2 of this proof are satisfied: let $a(r, i) = R(1 - ri)$, $a_n(r, i) = R(1 - ri(1 - 1/n))$, $b(r, i) = 1 - i$ and $b_n(r, i) = 1 - i(1 - 1/n)$; in particular $b_n \geq b \geq \varepsilon$.

If $i > 1/r$, then $\beta(r, i) = 0$ and since $\beta_n(r, i)$ is decreasing in i, we obtain

$$\begin{aligned}
&|\beta_n(r, i) - \beta(r, i)| \leq \frac{R(1 - r(1 - \frac{1}{n})\frac{1}{r})}{1 - (1 - \frac{1}{n})\frac{1}{r}} \\
&\leq R\frac{1/n}{1 - 1/r} \leq \frac{R}{\varepsilon n} \to 0.
\end{aligned} \tag{22}$$

Thus, $\beta_n(r, i)$ converges uniformly to $\beta(r, i)$. Since u is continuous, then $V_n^2(r, i)$ converges uniformly to $V^2(r, i)$.

4. Obvious. ■

Let

$$U(r) = \int_0^1 f(t)\left[t\alpha(t, k(r))u(r) + (1-t)u\left(\max\left\{0, \frac{R(1 - rt)}{1 - t}\right\}\right)\right] \mathrm{d}t, \tag{23}$$

and $k(r) = 1/r$.

Lemma 4 *Let $B > 0$, $x \in (0,1)$, $0 < \bar{t} < 1$ and $f \in \mathcal{F}_{\bar{t},B,x}$. Then, $U_n(r)$ converges uniformly to $U(r)$.*

Proof. We may write

$$U_n(r) = \int_0^{\bar{t}} f(t) V_n(r,t) \mathrm{d}t, \tag{24}$$

where

$$V_n(r,t) = \int_0^1 \left(t V_n^1(r,i) + (1-t) V_n^2(r,i) \right) \mathrm{d}p_{n-1,t}(i). \tag{25}$$

We may also define

$$V(r,t) = t\alpha(t,k(r))u(r) + (1-t)u\left(\max\left\{0, \frac{R(1-rt)}{1-t}\right\}\right)$$

and write $U(r) = \int_0^{\bar{t}} f(t)V(r,t)\mathrm{d}t$. Thus, in order to prove the lemma, it is enough to show that V_n converges uniformly to V for $r \in [1,R]$ and $t \in [0,\bar{t}]$.

Let $0 < \eta < 1 - \bar{t}$. Then,

$$|V_n(r,t) - V(r,t)| \leq \left| \int_0^{\bar{t}+\eta} V_n(r,i)\mathrm{d}p_{n-1,t}(i) - V(r,t) \right| + \left| \int_{\bar{t}+\eta}^1 V_n(r,i)\mathrm{d}p_{n-1,t}(i) \right|. \tag{26}$$

Since $p_{n-1,t}(B_\eta(t)) \geq 1 - \frac{t(1-t)}{(n-1)\eta^2}$ (see Freund [8, p. 190]), we have that

$$\begin{aligned} &\left| \int_{\bar{t}+\eta}^1 V_n(r,i)\mathrm{d}p_{n-1,t}(i) \right| \leq u(R)p_{n-1,t}([\bar{t}+\eta,1]) \leq \\ &u(R)p_{n-1,t}([0,t-\eta]\cup[t+\eta,1]) \leq u(R)\frac{t(1-t)}{(n-1)\eta^2} \leq u(R)\frac{1}{4(n-1)\eta^2}. \end{aligned} \tag{27}$$

Since $u(R)/[4(n-1)\eta^2]$ converges to zero and is independent of r and t, it remains to show that

$$\left| \int_0^{\bar{t}+\eta} V_n(r,i)\mathrm{d}p_{n-1,t}(i) - V(r,t) \right|$$

converges to zero uniformly. This follows from Lemma 2 and 3. ■

Let

$$WL(r) = \int_0^1 f(t)u\left(\max\left\{0, \frac{R(1-rt)}{1-t}\right\}\right)\mathrm{d}t \tag{28}$$

and

$$WR(r) = \int_0^1 f(t)\alpha(t, k(r))u(r)\mathrm{d}t. \tag{29}$$

Lemma 5 *Let $B > 0$, $x \in (0,1)$, $0 < \bar{t} < 1$ and $f \in \mathcal{F}_{\bar{t},B,x}$. Then, $WL_n(r)$ converges uniformly to $WL(r)$ and $WR_n(r)$ converges uniformly to $WR(r)$.*

Proof. Analogous to Lemma 4. ■

A.2 Analysis of the Limit Problem

Let $D = \{r \in [1,R] : U(r) \geq U(1)\}$ and $W = \{r \in [1,R] : WL(r) \geq WR(r)\}$.

Lemma 6 *The function U has a maximizer in $D \cap W$.*

Proof. Note that the set $D \cap W$ is compact, and non-empty, since $r = 1$ belongs to $D \cap W$. The function U is a continuous function of r. Hence, there exists r^* that maximizes U in $D \cap W$. ■

Lemma 7 *For all $B > 0$ and $x \in (0,1)$, there is $\tau \in [0,1]$ such that if $\tau < \bar{t} < 1$ and $f \in \mathcal{F}_{\bar{t},B,x}$ the following holds:*

There exists $\tilde{r} > 1/\bar{t}$ such that $U(\tilde{r}) > U(r)$ for all $1 \leq r \leq 1/\bar{t}$ and $WL(\tilde{r}) > WR(\tilde{r})$.

Lemma 7 is the key lemma to establish Proposition 1. Before presenting its proof, we need some technical lemmas.

Lemma 8 *$\int_0^r u' = u(r)$ for any $r > 0$.*

Proof. Let $0 < \delta < r$. Then $\int_\delta^r u' = u'(r) - u'(\delta)$ (see Wheeden and Zygmund [21, Theorem 5.52, p.83] and Rudin [18, Theorem 6.21, p. 134]).

Define $h_k = \chi_{[1/k,r]}u'$ for all $k \in \mathbb{N}$. Since $h_k \geq 0$ for all k, and $h_k \nearrow u'$, it follows that $\int_0^r h_k \to \int_0^r u'$. Hence, $\int_0^r u' = \lim_k[u(r) - u(1/k)] = u(r) - u(0) = u(r)$. ■

Let $g(r,t) = \frac{R(1-rt)}{1-t}$.

Lemma 9 *$\int_0^{1/r} u'(g(r,t))\mathrm{d}t = \int_0^R u'(y)\frac{R(r-1)}{(Rr-y)^2}\mathrm{d}y$ for all $r > 1$.*

Proof. We start by noting the following extension of the change of variable theorem (Rudin [18, Theorem 6.19, p. 132]): if $\gamma : [A, B] \to [b, a]$ is a strictly decreasing continuous function with γ' Riemann integrable and h is Riemann integrable on $[b, a]$, then $\int_b^a h(x)\mathrm{d}x = \int_A^B (-\gamma'(y))h(\gamma(y))\mathrm{d}y$.

Let $0 < \delta < 1/r$. Let $\gamma(y) = \frac{R-y}{Rr-y}$. Then $-\gamma(y) = \frac{R(r-1)}{(Rr-y)^2}$, and is therefore bounded by $1/R(r-1)$. Thus, by the above,

$$\int_0^\delta u' \circ g(r,t)\mathrm{d}t = \int_{\frac{R(1-r\delta)}{1-\delta}}^R u'(y)\frac{R(r-1)}{(Rr-y)^2}\mathrm{d}y. \tag{30}$$

Using an argument similar to that in Lemma 8, we can show that

$$\int_0^{1/r} u'(g(r,t))\mathrm{d}t = \lim_{\delta\to 1/r}\int_0^\delta u'(g(r,t))\mathrm{d}t = \int_0^R u'(y)\frac{R(r-1)}{(Rr-y)^2}\mathrm{d}y. \tag{31}$$

■

We can now proceed with the proof of Lemma 7.

Proof of Lemma 7. Since $WL(1) = u(R) > u(1) = WR(1)$, then there exists $\zeta > 0$ such that $1 < r < 1 + \zeta$ implies $WL(r) > WR(r)$.

Let f be such that $\int_0^1 tf(t)\mathrm{d}t = x$. Consider the following function $M : [0, 1] \to \mathbb{R}$ defined by

$$M(r) = \int_0^1 [u'(r) - Ru'(g(r,t))]tf(t)\mathrm{d}t. \tag{32}$$

Clearly, $M(1) = [u'(1) - Ru'(R)]\int_0^1 tf(t)\mathrm{d}t = [u'(1) - Ru'(R)]x$ and so $M(1) > 0$ (see Diamond and Dybvig [6, footnote 2]).

Let $\{\bar{t}_k\}_k \subset (0, 1)$ be such that $\lim_k \bar{t}_k = 1$ and $\{f_k\}$ be a sequence of densities belonging to $\mathcal{F}_{\bar{t}_k,B,x}$, but otherwise arbitrary. Let $\bar{r}_k = 1/\bar{t}_k$, for all k.

Claim 1 $\lim_k \int_0^1 [u'(\bar{r}_k) - Ru'(g(\bar{r}_k, t))]tf_k(t)\mathrm{d}t = [u'(1) - Ru'(R)]x$.

Proof. Since u' is continuous, we have that $u'(\bar{r}_k)x \to u'(1)x$; thus, it remains to show that $\int_0^1 u' \circ g(\bar{r}_k, t)tf_k(t)\mathrm{d}t \to xu'(R)$. Note that $xu'(R) = \int_0^1 u' \circ g(1, t)tf_k(t)\mathrm{d}t$ for all k. We have that $u' \circ g(\bar{r}_k, t)tf_k(t) \geq u'(R)$ for all $t \in [0, \bar{t}_k]$ and so

$$\begin{aligned} &\left|\int_0^1 u' \circ g(\bar{r}_k, t)tf_k(t)\mathrm{d}t - \int_0^1 u' \circ g(1, t)tf_k(t)\mathrm{d}t\right| \\ &\leq B\int_0^1 (u' \circ g(\bar{r}_k, t) - u' \circ g(1, t))\chi_{[0,\bar{t}_k]}\mathrm{d}t. \end{aligned} \tag{33}$$

Since $\int_0^1 u' \circ g(1,t))\chi_{[0,\bar{t}_k]}\mathrm{d}t \to u'(R) = \int_0^1 u' \circ g(1,t)\mathrm{d}t$, it is enough to show that

$$\int_0^{\bar{t}_k} u' \circ g(\bar{r}_k, t)\mathrm{d}t = \int_0^1 u' \circ g(\bar{r}_k, t)\chi_{[0,\bar{t}_k]}\mathrm{d}t \to \int_0^1 u' \circ g(1,t)\mathrm{d}t.$$

Clearly, $0 \le u' \circ g(\bar{r}_k, t)\chi_{[0,\bar{t}_k]} \to u' \circ g(1,t)$. Let $\varepsilon > 0$. Then, by the bounded convergence theorem (Wheeden and Zygmund [21, Corollary 5.37, p. 76]) $\int_0^{1-\varepsilon} u' \circ g(\bar{r}_k, t)\mathrm{d}t \to u'(R)(1-\varepsilon)$. By Lemma 9, for k large,

$$\int_{1-\varepsilon}^1 u' \circ g(\bar{r}_k, t)\chi_{[0,\bar{t}_k]}\mathrm{d}t = \int_{1-\varepsilon}^{\bar{t}_k} u' \circ g(\bar{r}_k, t)\mathrm{d}t = \int_0^{R(1-\bar{r}_k+\varepsilon\bar{r}_k)/\varepsilon} u'(y)\frac{R(\bar{r}_k - 1)}{(R\bar{r}_k - y)^2}\mathrm{d}y. \tag{34}$$

We have that

$$\begin{aligned}
&\int_0^{R(1-\bar{r}_k+\varepsilon\bar{r}_k)/\varepsilon} u'(y)\frac{R(\bar{r}_k - 1)}{(R\bar{r}_k - y)^2}\mathrm{d}y \le \\
&\frac{R(\bar{r}_k - 1)}{(R\bar{r}_k - 1)^2}\int_0^1 u'(y)\mathrm{d}y + u'(1)R(\bar{r}_k - 1)\int_1^{R(1-\bar{r}_k+\varepsilon\bar{r}_k)/\varepsilon} \frac{1}{(R\bar{r}_k - y)^2}\mathrm{d}y = \\
&u(1)\frac{R(\bar{r}_k - 1)}{(R\bar{r}_k - 1)^2} + u'(1)\left[\frac{R(\bar{r}_k - 1)}{R\bar{r}_k - R(1-\bar{r}_k+\varepsilon\bar{r}_k)/\varepsilon} - \frac{R(\bar{r}_k - 1)}{R\bar{r}_k - 1}\right] = \\
&u(1)\frac{R(\bar{r}_k - 1)}{(R\bar{r}_k - 1)^2} + u'(1)\left[\varepsilon - \frac{R(\bar{r}_k - 1)}{R\bar{r}_k - 1}\right] \to_{k\to\infty} \\
&\varepsilon u'(1).
\end{aligned} \tag{35}$$

Thus,

$$u'(R) \le \liminf_k \int_0^{\bar{t}_k} u' \circ g(\bar{r}_k, t)\mathrm{d}t \le \limsup_k \int_0^{\bar{t}_k} u' \circ g(\bar{r}_k, t)\mathrm{d}t \le \varepsilon u'(1) + (1-\varepsilon)u'(R), \tag{36}$$

and so $\lim_k \int_0^{\bar{t}_k} u' \circ g(\bar{r}_k, t)\mathrm{d}t = u'(R)$. ■

Hence, it follows from claim 1 that there exists $0 < \tau < 1$ such that if $\tau < \bar{t} < 1$ and $f \in \mathcal{F}_{\bar{t},B,x}$, then $M_f(\bar{r}) > M_f(1)$ and $WL(r) > WR(r)$ for all $r \in [1, \bar{r}]$.

Let $\bar{t} \in (\tau, 1)$ and $f \in \mathcal{F}_{\bar{t},B,x}$. Note that U is concave in $[0, \bar{r}]$, where $\bar{r} = 1/\bar{t}$ as before. This follows from the fact that both u and $r \mapsto u \circ g(r,t)$ are concave for all $t \in [0, \bar{r}]$: we have that $\partial u \circ g(r,t)/\partial r = u'' \circ g(r,t)(tR)^2/(1-t)^2 < 0$.

Claim 2 *$U'(r) = M(r)$ for all $r \in [1, \bar{r}]$.*

Proof. Let $r \in (1, \bar{r}]$ and let $r_k \nearrow r$. Then

$$\frac{U(r_k) - U(r)}{r_k - r} = \int_0^{\bar{t}} f(t) \left[t\frac{u(r_k) - u(r)}{r_k - r} + (1-t)\frac{u \circ g(r_k, t) - u \circ g(r, t)}{r_k - r} \right] \mathrm{d}t. \tag{37}$$

So, it is enough to show that

$$\int_0^{\bar{t}} f(t)t\frac{u(r_k) - u(r)}{r_k - r}\mathrm{d}t \to \int_0^{\bar{t}} f(t)u'(r)\mathrm{d}t,$$

and

$$\int_0^{\bar{t}} f(t)(1-t)\frac{u \circ g(r_k, t) - u \circ g(r, t)}{r_k - r}\mathrm{d}t \to \int_0^{\bar{t}} f(t)tRu' \circ g(r, t)\mathrm{d}t.$$

We have that $\{(u(r_k) - u(r))/(r_k - r)\}_k$ is non-negative and decreasing and that $t \mapsto tf(t)(u(r_1) - u(r))/(r_1 - r)$ is integrable. The desired convergence then follows from the monotone convergence theorem (see Wheeden and Zygmung [21, Theorem 5.32, p. 75]). Similarly, $\{(u \circ g(r_k, t) - u \circ g(r, t))/(r_k - r)\}_k$ is non-positive and decreasing, and so $\{-(u \circ g(r_k, t) - u \circ g(r, t))/(r_k - r)\}_k$ is non-negative and increasing, and so the desired convergence follows also from the monotone convergence theorem.

The case $r \in [1, \bar{r})$ and $r_k \searrow r$ is analogous. ■

Since U is concave, then M is decreasing. This implies that $U(\bar{r}) \geq U(r)$ for all $1 \leq r \leq \bar{r}$ as follows: A necessary condition for a solution to the problem

$$\begin{aligned} &\max_{r \in [1, R]} U(r) \\ &\text{subject to } r \in W \cap D \\ &\text{and } r \leq \bar{r}. \end{aligned} \tag{38}$$

is that

$$[r - \bar{r}]M(r) = 0; \tag{39}$$

Since $M(r) > 0$ for all $r \in [1, \bar{r}]$, it follows that $\bar{r}$ maximizes $U(r)$ in $[1, \bar{r}]$.

Finally, we claim that there exists $\tilde{r} > \bar{r}$ such that $U(\tilde{r}) > U(\bar{r})$ and $WL(\tilde{r}) > WR(\tilde{r})$.

Since $WL(\bar{r}) > WR(\bar{r})$, we conclude that there exists a ball $B(\bar{r})$ around $\bar{r}$ such that $r \in B(\bar{r})$ implies $WL(r) > WR(r)$. Therefore, to prove the existence of $\tilde{r}$ with the above properties, it is enough to show that

$$\lim_{r \searrow \bar{r}} \frac{U(r) - U(\bar{r})}{r - \bar{r}} > 0. \tag{40}$$

This is so, because if equation (40) holds, then it cannot be the case that $\frac{U(r)-U(\bar{r})}{r-\bar{r}} \leq 0$ for all $r > \bar{r}$ in the ball $B(\bar{r})$ around $\bar{r}$. This implies the existence of $\tilde{r} > \bar{r}$ in $B(\bar{r})$ such that $\frac{U(\tilde{r})-U(\bar{r})}{\tilde{r}-\bar{r}} > 0$; this, of course, implies that $U(\tilde{r}) > U(\bar{r})$.

We have that

$$\begin{aligned}
&\frac{U(r) - U(\bar{r})}{r - \bar{r}} = \\
&\int_0^{1/r} \left[t\frac{u(r) - u(\bar{r})}{r - \bar{r}} + (1-t)\frac{u \circ g(r,t) - u \circ g(\bar{r},t)}{r - \bar{r}} \right] f(t)\mathrm{d}t + \\
&\frac{1}{r - \bar{r}} \int_{1/r}^{\bar{t}} \{t[\alpha(t,k(r))u(r) - u(\bar{r})] - (1-t)u \circ g(\bar{r},t)\} f(t)\mathrm{d}t.
\end{aligned} \tag{41}$$

Let $\{r_k\}_k$ be such that $r_k \searrow \bar{r}$. Note that $\alpha(t,k(r_k)) = 1/(tr_k) = t_k/t$, with $t_k = 1/r_k$. Let $\varepsilon > 0$; if $k \in \mathbb{N}$ is sufficiently large, then we have that $|t_k u(r_k) - tu(\bar{r})| < \varepsilon$ for all $t \in [t_k, \bar{t}]$ since

$$\begin{aligned}
|t_k u(r_k) - tu(\bar{r})| &\leq t_k|u(r_k) - u(\bar{r})| + u(\bar{r})|t_k - t| \\
&\leq \bar{t}|u(r_k) - u(\bar{r})| + u(\bar{r})|t_k - \bar{t}|,
\end{aligned} \tag{42}$$

and $t_k \to \bar{t}$. Therefore,

$$\begin{aligned}
&\left| \frac{1}{r_k - \bar{r}} \int_{1/r_k}^{\bar{t}} \{t[\alpha(t,k(r_k))u(r_k) - u(\bar{r})] f(t)\mathrm{d}t \right| = \\
&\left| \frac{1}{r_k - \bar{r}} \int_{1/r_k}^{1/\bar{r}} [t_k u(r_k) - tu(\bar{r})] f(t)\mathrm{d}t \right| \leq \\
&\frac{\varepsilon}{r_k - \bar{r}} \int_{1/r_k}^{1/\bar{r}} f(t)\mathrm{d}t = \varepsilon \frac{F(1/\bar{r}) - F(1/r_k)}{r - \bar{r}} \to \varepsilon \frac{f(1/\bar{r})}{\bar{r}^2}.
\end{aligned} \tag{43}$$

Since ε is arbitrary, then

$$\lim_k \frac{1}{r_k - \bar{r}} \int_{1/r_k}^{\bar{t}} \{t[\alpha(t,k(r))u(r) - u(\bar{r})] f(t)\mathrm{d}t = 0. \tag{44}$$

Also, by the fundamental theorem of calculus,

$$\frac{1}{r_k-\bar{r}}\int_{1/r_k}^{1/\bar{r}}(1-t)f(t)u\circ g(\bar{r},t)\mathrm{d}t \to (1-\bar{t})f(\bar{t})u\circ g(\bar{r},\bar{t})/\bar{r}^2 = 0, \tag{45}$$

since $g(\bar{r},\bar{t})=0$ and $u(0)=0$. Hence, it remains to show that

$$\lim_{r\searrow\bar{r}}\int_0^{1/r}\left[t\frac{u(r)-u(\bar{r})}{r-\bar{r}}+(1-t)\frac{u\circ g(r,t)-u\circ g(\bar{r},t)}{r-\bar{r}}\right]f(t)\mathrm{d}t>0. \tag{46}$$

Defining

$$h_k(t)=\begin{cases}\left[t\frac{u(r_k)-u(\bar{r})}{r_k-\bar{r}}+(1-t)\frac{u\circ g(r_k,t)-u\circ g(\bar{r},t)}{r_k-\bar{r}}\right]f(t) & \text{if } t\in[0,1/r_k]\\ 0 & \text{otherwise,}\end{cases} \tag{47}$$

we see that $\lim_k h_k(t)\to[u'(\bar{r})-Ru'\circ g(\bar{r},t)]tf(t)\chi_{[0,1/\bar{r})}$. Thus, by Claim 3 and the Bounded Convergence Theorem, we obtain

$$\begin{aligned}&\lim_{r\searrow\bar{r}}\int_0^{1/r}\left[t\frac{u(r)-u(\bar{r})}{r-\bar{r}}+(1-t)\frac{u\circ g(r,t)-u\circ g(\bar{r},t)}{r-\bar{r}}\right]f(t)\mathrm{d}t=\\&\lim_{k\to\infty}\int_0^1 h_k(t)\mathrm{d}t=\int_0^1[u'(\bar{r})-Ru'\circ g(\bar{r},t)]tf(t)\mathrm{d}t=M(\bar{r})>0.\end{aligned} \tag{48}$$

Thus, it remains only to show that the sequence of functions $\{h_k\}$ is bounded, which is done in the following claim.

Claim 3 *There exists $M\in\mathbb{R}$ such that $|h_k|\leq M$ for all $k\in\mathbb{N}$.*

Proof. Let

$$h_k^1=t\frac{u(r_k)-u(\bar{r})}{r_k-\bar{r}}\chi_{[0,1/r_k]}$$

and

$$h_k^2=(1-t)\frac{u\circ g(r_k,t)-u\circ g(\bar{r},t)}{r_k-\bar{r}}\chi_{[0,1/r_k]}.$$

Obviously, $|h_k|\leq|h_k^1|+|h_k^2|$.

Since u is concave, then

$$\left\{\frac{u(r_k)-u(\bar{r})}{r_k-\bar{r}}\right\}_k$$

increases. Thus $|h_k^1(t)| \leq tu'(\bar{r}) \leq u'(\bar{r})$.

Define

$$\phi_k(t) = u \circ g(r_k, t) - u \circ g(\bar{r}, t), \tag{49}$$

for $t \in [0, 1/r_k]$ (note that $\phi_k(t) = (r_k - \bar{r})h_k^2(t)$). Then,

$$\phi_k' = \frac{R}{(1-t)^2} \left[u' \circ g(\bar{r}, t)(\bar{r} - 1) - u' \circ g(r_k, t)(r_k - 1) \right]. \tag{50}$$

Since $0 < \bar{r} - 1 < r_k - 1$ and $0 < u' \circ g(\bar{r}, t) < u' \circ g(r_k, t)$, then $\phi_k' < 0$. Thus, $h_k^2(1/r_k) \leq h_k(t) \leq 0$ for all $t \in [0, 1/r_k]$.

Consider

$$|h_k^2(1/r_k)| = \frac{u\left(\frac{R(r_k - \bar{r})}{r_k - 1}\right)}{r_k - \bar{r}}.$$

By the mean value theorem, we have that

$$|h_k^2(1/r_k)| = u'\left(\frac{R(c_k - \bar{r})}{c_k - 1}\right) \frac{R(c_k - \bar{r})}{(c_k - 1)^2}, \tag{51}$$

for $\bar{r} \leq c \leq r_k$. Note that $r_k - \bar{r} \leq R(r_k - \bar{r})/(r_k - 1)$ and so $u'(r_k - \bar{r}) \geq u'(R(r_k - \bar{r})/(r_k - 1))$. Since $\lim_{c \to 0} cu'(c) \in \mathbb{R}$, there exists $m \in \mathbb{R}$ such that $\lim_k u'(r_k - \bar{r})(r_k - \bar{r}) \leq m$. Then, $\limsup_k u'(R(r_k - \bar{r})/(r_k - 1)) \leq \lim_k u'(r_k - \bar{r})(r_k - \bar{r}) \leq m$; hence $\{|h_k^2(1/r_k)|\}$, and therefore $|h_k^2|$, is bounded by $Rm/(\bar{r} - 1)$. ■

This completes the proof. ■

A.3 Existence of a Non-Autarkic Equilibrium with a Finite Number of Consumers

In this section we establish the existence of an equilibrium in which all consumers deposit at the bank.

Lemma 10 *For any n, U_n is upper semi-continuous.*

Proof. Note that

$$k_n(r) = \begin{cases} 1 & \text{if } r = 1 \\ \frac{n-j}{n} & \text{if } \frac{n}{n-j+1} < r \leq \frac{n}{n-j}, \quad j = 1, \ldots, n-1 \\ 0 & \text{if } r > n \end{cases} \tag{52}$$

Thus, k_n is upper semi-continuous, and so is U_n. ■

Lemma 11 *The program*

$$\begin{aligned}&\max_{r\in[1,R]} U_n(r)\\ &\textit{subject to } r \in W_n \cap D_n,\end{aligned} \tag{53}$$

has a solution.

Proof. Let $I_j = [\frac{n}{n-j+1}, \frac{n}{n-j}]$ for $j = 1, \ldots, n-1$, $I_n = [n, \infty)$ and $I_0 = \{1\}$. For each $j = 0, \ldots, n$, consider the following problem, denoted P^j:

$$\begin{aligned}&\max_r U_n^j(r) = E_t E_i \left[\frac{n-j}{(n-1)i+1} u(r) + (1-t) u\left(\max\left\{\frac{R(n-r(n-1)i}{n-(n-1)i}\right\}\right)\right]\\ &\text{subject to } r \in I_j \cap W_n^j \cap D_n,\end{aligned} \tag{54}$$

where

$$W_n^j = \{r \in [1, R] : WL(r) \geq \frac{n-j}{(n-1)i+1} u(r)\}, \tag{55}$$

for $j = 1, \ldots, n$ and $W_n^0 = W_n$. Let S be the set of those js for which the constraint set is non-empty; the set S is non-empty since $1 \in I_1 \cap W_n^0 \cap D_n$ and so $0 \in S$. Then, if $j \in S$, the above problem has a solution, since the objective function is upper semi-continuous, and the constraint set is compact. Let r_j^* be a solution to P^j.

Since U_n jumps down if (and only if) $r \in \{1, \frac{n}{n-1}, \frac{n}{n-2}, \ldots, n\}$, it follows that $\frac{n}{n-j}$ is a solution to P^j, $j > 0$ then $U_n^{j-1}(r_{j-1}^*) \geq U_n^{j-1}(\frac{n}{n-j}) = U_n(\frac{n}{n-j}) > U_n^j(\frac{n}{n-j})$. Thus, in order to find a maximum in $[1, R]$ we can concentrate on those js for which $\frac{n}{n-j}$ is not a solution to P^j; let J be such a set. Let r^* be a solution to $\max_{r\in\{r_j^*:j\in J\}} U_n(r)$.

We claim that r^* solves the problem (53). Letting $\tilde{I}_j = (\frac{n}{n-j+1}, \frac{n}{n-j}]$ for $j = 1, \ldots, n-1$, $\tilde{I}_n = (n, \infty)$ and $\tilde{I}_0 = \{1\}$, we can separate it into the following problems, denoted $\tilde{P}^j$:

$$\begin{aligned}&\max_r U_n^j(r) = E_t E_i \left[\frac{n-j}{(n-1)i+1} u(r) + (1-t) u\left(\max\left\{\frac{R(n-r(n-1)i}{n-(n-1)i}\right\}\right)\right]\\ &\text{subject to } r \in \tilde{I}_j \cap W_n^j \cap D_n.\end{aligned} \tag{56}$$

Since for $j \in J$ the solution does not involve $\frac{n}{n-j}$, $\tilde{P}^j$ has the same solutions as P^j. Hence, r^* is a solution to program (53). ■

Lemma 12 *For all n, there is a symmetric equilibrium in which all consumers deposit.*

Proof. The strategies are: the bank offers r^*, and the consumers choose

$$d^*(r) = \begin{cases} 1 & \text{if } r \in D_n \cap W_n \\ 0 & \text{otherwise,} \end{cases} \tag{57}$$

$$w^*(r,d,2) = \begin{cases} 0 & \text{if } r \in D_n \cap W_n \text{ and } d = 1 \\ 1 & \text{otherwise,} \end{cases} \tag{58}$$

and $w^*(r,d,1) = 1$ for all (r,d). ■

A.4 A Positive Probability of an Equilibrium Bank Run

In this section we prove Proposition 1. By Corollary 12, there is an equilibrium in which all consumers deposit for all $n \in \mathbb{N}$ and all functions f.

Let τ be as in Lemma 7 and let $\bar{t} > \tau$ and $f \in \mathcal{F}_{\bar{t}}$. Then, there is $\tilde{r} > 1/\bar{t}$ such that $U(\tilde{r}) > U(r)$ for all $r \in [1, 1/\bar{t}]$ and $WL(\tilde{r}) > WR(\tilde{r})$. Then, by Lemmas 4 and 5, it follows that $U_n(\tilde{r}) > U_n(r)$ for all $r \in [1, 1/\bar{t}]$ and $WL_n(\tilde{r}) > WR_n(\tilde{r})$ if n is large. Thus, $r_n^* > 1/\bar{t}$. Hence, the bank fails with a probability given by

$$\int_0^{\bar{t}} f(t) \left(\sum_{i \in S_n : i > 1/r_n^*} p_{n,t}(i) \right) \mathrm{d}t > 0, \tag{59}$$

since $\sum_{i \in S_n : i > 1/r^*} p_{n,t}(i) > 0$ if $t > 0$.

A.5 A Limit Result on the Probability of an Equilibrium Bank Run

Here we prove proposition 2. First, we claim that the sequence of interest rates is bounded away from $\bar{r}$.

Claim 4 *There exists $\varepsilon > 0$ such that $r_n^* \geq \bar{r} + \varepsilon$ for all n.*

Proof. Suppose not. Then there is a subsequence $\{r^*_{n_k}\}$ such that $r^*_{n_k} \to \bar{r}$. Let $\tilde{r}$ be as in Lemma 7. Then, $U_{n_k}(r^*_{n_k}) \geq U_{n_k}(\tilde{r})$ for all k and Lemma 4 imply that $U(\bar{r}) \geq U(\tilde{r})$, a contradiction. ■

Hence,

$$\left\{ i > \frac{1}{\bar{r}+\varepsilon} \right\} \subseteq \left\{ i > \frac{1}{r^*_n} \right\},$$

and so

$$\gamma_n = \int_0^{\bar{t}} f(t) \left(\sum_{i \in S_n : i > 1/r^*_n} p_{n,t}(i) \right) \mathrm{d}t \geq \int_0^{\bar{t}} f(t) \left(\int_0^1 \chi_{(1/(\bar{r}+\varepsilon),1]} \mathrm{d}p_{n,t} \right) \mathrm{d}t. \tag{60}$$

Let $\delta > 0$ and $g : [0,1] \to [0,1]$ be a continuous function satisfying $g = \chi_{(1/(\bar{r}+\varepsilon),1]}$ in $[0, 1/(\bar{r}+\varepsilon)] \cup [1/(\bar{r}+\varepsilon)+\delta, 1]$. Then,

$$\int_0^{\bar{t}} f \left(\int_0^1 g \mathrm{d}p_{n,t} \right) \mathrm{d}t \to \int_0^{\bar{t}} fg \geq 1 - F(1/(\bar{r}+\varepsilon)+\delta). \tag{61}$$

Since $g \leq \chi_{(1/(\bar{r}+\varepsilon),1]}$, we obtain

$$\liminf_n \int_0^{\bar{t}} f(t) \left(\int_0^1 \chi_{(1/(\bar{r}+\varepsilon),1]} \mathrm{d}p_{n,t} \right) \mathrm{d}t \geq 1 - F(1/(\bar{r}+\varepsilon)+\delta); \tag{62}$$

since this holds for all $\delta > 0$, it follows that

$$\liminf_n \int_0^{\bar{t}} f(t) \left(\int_0^1 \chi_{(1/(\bar{r}+\varepsilon),1]} \mathrm{d}p_{n,t} \right) \mathrm{d}t \geq 1 - F(1/(\bar{r}+\varepsilon)), \tag{63}$$

by letting $\delta \to 0$. Hence,

$$\liminf_n \gamma_n \geq 1 - F(1/(\bar{r}+\varepsilon)) > 0, \tag{64}$$

as desired.

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